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PLANE PROBLEM OF VIBRATIONS OF AN ELASTIC FLOATING PLATE UNDER PERIODIC EXTERNAL LOADING

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The Wiener–Hopf technique is used to construct an analytical solution of the problem of vibrations of a semi-infinite elastic floating plate under periodic external loading. The solution is obtained in explicit form ignoring draft. The dependences of the amplitudes of surface waves and ice-plate deflection on the loading distribution and frequency, ice thickness, and liquid depth are studied numerically. It is established that for some types of acting load, no waves propagate in the plate and liquid and the plate vibrations are standing waves localized near the loading region. An example of such vibrations is given and a condition for the occurrence of localized vibrations is found.

Key words: *surface waves, flexural-gravity waves, elastic thin plate, Wiener–Hopf technique, localized vibrations.*

At present, the problem of the hydroelastic behavior of a plate floating on a liquid surface is of interest in connection with the design of floating platforms of various applications: artificial islands, airdromes, launching sites, etc. The huge dimensions of such objects prevents fulfillment of similarity criteria in experimental studies; therefore, numerical modeling plays an important role is studies of such objects.

The diffraction of surface waves on a floating elastic plate has been studied fairly well. The dynamics of an infinite floating plate under external loading has also been examined in detail using integral transformations (see, e.g., [1–3]). However, the behavior of a floating elastic finite plate under dynamic loading has been investigated inadequately. Numerical solutions of these problems in plane and three-dimensional formulations were constructed using an expansion in normal modes and the Galerkin or Rayleigh–Ritz methods [4–7]. In the present paper, an analytical solution of the present problem for a semi-infinite plate is constructed using the Wiener–Hopf technique.

1. Formulation of the Problem. It is assumed that the liquid is ideal and incompressible of depth H_0 and that its flow is irrotational. The examined plate has constant thickness h and its vibrations are caused by a time-periodic external pressure applied to the plate surface. The problem is solved in a plane formulation. The plate edge is the origin of Cartesian coordinates Oxy. The plate covers the liquid surface at $x > 0$, and the remaining part of the liquid surface is free. It is assumed that the plate thickness is much smaller than the length of the waves propagating in the plate. A thin plate model is used.

The liquid-velocity potential φ satisfies the Laplace equation and the boundary conditions

$$\Delta\varphi = 0 \qquad (-H_0 < y < 0),$$

$$\varphi_y = 0 \quad (y = -H_0), \qquad \varphi_y = w_t \quad (y = 0),$$

$$D\,\frac{\partial^4 w}{\partial x^4} + \rho_0 h\,\frac{\partial^2 w}{\partial t^2} = p + q(x)\,\mathrm{e}^{-i\omega t} \qquad (y = 0, \quad x > 0), \tag{1.1}$$

$$p = -\rho(\varphi_t + gw), \qquad \varphi_t + gw = 0 \quad (y = 0,\ x < 0).$$

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Here w is the vertical displacement of the upper liquid surface (plate), p is the hydrodynamic pressure, $q(x)$ is the external loading intensity, g is the acceleration due to gravity, D is the cylindrical stiffness of the plate, ρ and ρ_0 are the densities of the liquid and plate, and t is time. On the plate edge, the moment and the shear force should be equal to zero:

$$\frac{\partial^2 w}{\partial x^2} = \frac{\partial^3 w}{\partial x^3} = 0 \qquad (y = 0, \quad x = 0). \tag{1.2}$$

We first consider the case of a concentrated load: $q(x) = q_0\delta(x - x_0)$. The time dependences of all functions are expressed in terms of the factor $\mathrm{e}^{-i\omega t}$. We introduce the characteristic length $l = g/\omega^2$ and the dimensionless variables

$$x' = x/l, \qquad y' = y/l, \qquad \varphi' = \varphi\omega\rho/q_0, \qquad w' = w\rho g/q_0, \qquad t' = \omega t.$$

Below, the primes are omitted. The potential is represented as $\varphi = \phi\,\mathrm{e}^{-it}$. Then from (1.1) and (1.2), we obtain the following boundary-value problem for ϕ:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \qquad (-H < y < 0);$$

$$\frac{\partial \phi}{\partial y} = 0 \qquad (y = -H); \tag{1.3}$$

$$\frac{\partial \phi}{\partial y} - \phi = 0 \qquad (y = 0, \quad x < 0); \tag{1.4}$$

$$\Big(\beta\,\frac{\partial^4}{\partial x^4} + 1 - \delta\Big)\frac{\partial \phi}{\partial y} - \phi = -i\delta(x - x_*) \qquad (y = 0, \quad x > 0); \tag{1.5}$$

$$\frac{\partial^2}{\partial x^2}\,\frac{\partial \phi}{\partial y} = \frac{\partial^3}{\partial x^3}\,\frac{\partial \phi}{\partial y} = 0 \qquad (y = 0, \quad x = 0); \tag{1.6}$$

$$H = H_0/l, \qquad \beta = D/(\rho g l^4), \qquad x_* = x_0/l, \qquad \delta = \rho_0 h/(\rho l).$$

The dimensionless parameters of the problem are as follows: the liquid depth H, the point of application of the external load x_*, the normalized stiffness β, and the normalized mass per length of the plate δ. In addition, the radiation conditions for $|x| \to \infty$ and the regularity condition near the edge (local boundedness of energy) should be satisfied.

2. Integral Equations. The problem is solved using the Wiener–Hopf technique in the Jones interpretation [8]. We introduce the following functions of the complex variable α:

$$\Phi_+(\alpha, y) = \int_0^\infty \mathrm{e}^{i\alpha x}\,\phi(x, y)\,dx, \qquad \Phi_-(\alpha, y) = \int_{-\infty}^0 \mathrm{e}^{i\alpha x}\,\phi(x, y)\,dx,$$

$$\Phi(\alpha, y) = \Phi_-(\alpha, y) + \Phi_+(\alpha, y). \tag{2.1}$$

The function $\Phi_+(\alpha, y)$ is defined in the upper half plane $\operatorname{Im}\alpha > 0$, and the function $\Phi_-(\alpha, y)$ in the lower semiplane $\operatorname{Im}\alpha < 0$. By means of analytic continuation, these functions can be defined in the entire complex plane. The function $\Phi(\alpha, y)$ is a Fourier image for the function $\phi(x, y)$ and satisfies the equation $\partial^2\Phi/\partial y^2 - \alpha^2\Phi = 0$. The general solution of this equation subject to the condition at the bottom (1.3) has the form

$$\Phi(\alpha, y) = C(\alpha)\cosh\,(\alpha(y + H))/\cosh\,(\alpha H). \tag{2.2}$$

We consider dispersion relations for surface and flexural-gravity waves in a liquid of finite depth. The periodic solutions of the Laplace equation subject to the condition at the bottom (1.3) has the form $\mathrm{e}^{i\alpha x}\cosh\,(\alpha(y + H))/\cosh\,(\alpha H)$. For surface waves, the values of α should satisfy the dispersion relation

$$K_1(\alpha) \equiv \alpha\tanh\,(\alpha H) - 1 = 0,$$

which has two real roots $\pm\gamma$, and an countable set of purely imaginary roots $\pm\gamma_n$ $(n = 1, 2, \ldots)$ symmetric about the real axis [9]; $\gamma_n \to in\pi/H$ as $n \to \infty$.

For the waves propagating in the liquid under the plate (flexural-gravity waves), we obtain the dispersion relation

$$K_2(\alpha) \equiv (\beta\alpha^4 + 1 - \delta)\alpha \tanh(\alpha H) - 1 = 0,$$

which has two real roots $\pm\alpha_0$, and an countable set of purely imaginary roots $\pm\alpha_n$ $(n = 1, 2, \ldots)$ symmetric about the real axis, and four complex roots symmetric about the real and imaginary axes [9]. We denote the root lying in the first quadrant by α_{-1} and the root in the second quadrant by α_{-2}, $\alpha_n \to in\pi/H$ as $n \to \infty$.

The dispersion functions $K_1(\alpha)$ and $K_2(\alpha)$ are even. The real roots of the dispersion relations define propagating waves and the remaining roots define edge waves, which decay exponentially away from the plate edge.

Let us examine the behavior of the functions $\Phi_\pm(\alpha, y)$. For $x \to -\infty$, the potential is a wave of the form $R\,\mathrm{e}^{-i\gamma x}$ and a set of exponentially decaying waves. The least rapidly decaying wave corresponds to the root γ_1. Therefore, $\Phi_-(\alpha, y)$ is analytic in the semiplane $\operatorname{Im}\alpha < |\gamma_1|$, except for the pole at $\alpha = -\gamma$. For $x \to \infty$, the potential φ_1 is a propagating wave of the form $T\,\mathrm{e}^{i\alpha_0 x}$ and a set of exponentially decaying modes. Therefore, the function $\Phi_+(\alpha, y)$ is analytic in the semiplane $\operatorname{Im}\alpha > -c$, except for the pole at the point $\alpha = \alpha_0$, where $c = \min\{|\alpha_1|, \operatorname{Im}(\alpha_{-1})\}$.

Let $D_\pm(\alpha)$ designate integrals of the form (2.1) with the integrand function ϕ replaced by the left side of boundary condition (1.4), and let $F_\pm(\alpha)$ designate similar expressions in which the integrand is the left side of expression (1.5). We introduce the functions

$$D(\alpha) = D_-(\alpha) + D_+(\alpha), \qquad F(\alpha) = F_-(\alpha) + F_+(\alpha).$$

The functions $D(\alpha)$ and $F(\alpha)$ are Fourier transforms of the dispersion functions, which have the meaning of generalized functions [10]. For them, the following relations hold:

$$D(\alpha) = C(\alpha)K_1(\alpha), \qquad F(\alpha) = C(\alpha)K_2(\alpha).$$

From boundary conditions (1.4) and (1.5), we have

$$\begin{gathered} D_-(\alpha) = 0, \qquad D_+(\alpha) = C(\alpha)K_1(\alpha), \\ F_+(\alpha) = -i\,\mathrm{e}^{i\alpha x_*}, \qquad F_-(\alpha) - i\,\mathrm{e}^{i\alpha x_*} = C(\alpha)K_2(\alpha). \end{gathered} \tag{2.3}$$

Elimination of $C(\alpha)$ yields the equation

$$(F_-(\alpha) - i\,\mathrm{e}^{i\alpha x_*})K(\alpha) = D_+(\alpha), \qquad K(\alpha) = K_1(\alpha)/K_2(\alpha). \tag{2.4}$$

According to the Wiener–Hopf technique, it is necessary to factorize the function $K(\alpha)$, i.e., to represent it as

$$K(\alpha) = K_+(\alpha)K_-(\alpha),$$

where the functions $K_\pm(\alpha)$ are regular in the same regions as the functions $\Phi_\pm(\alpha, y)$. The function $K(\alpha)$ has zeros and poles at the points $\pm\gamma$ and $\pm\alpha_0$, respectively, on the real axis. We therefore consider the analyticity regions S_+ and S_- (S_+ is the semiplane $\operatorname{Im}\alpha > -c$ with cuts eliminating the points α_0 and γ and S_- is the semiplane $\operatorname{Im}\alpha < |\gamma_1|$ with cuts eliminating the points $-\alpha_0$ and $-\gamma$).

We introduce the function

$$g(\alpha) = K(\alpha)\beta(\alpha^2 - \alpha_0^2)(\alpha^2 - \alpha_{-1}^2)(\alpha^2 - \alpha_{-2}^2)/(\alpha^2 - \gamma^2).$$

The function $g(\alpha)$ on the real axis has no zeros, is bounded, and tends to unity at infinity. We factorize $g(\alpha)$ as follows [8]:

$$g(\alpha) = g_+(\alpha)g_-(\alpha), \qquad g_\pm(\alpha) = \exp\Big[\pm\frac{1}{2\pi i}\int\limits_{-\infty\mp i\sigma}^{\infty\mp i\sigma} \frac{\ln g(x)}{x - \alpha}\,dx\Big], \qquad \sigma < |\gamma_1|, c.$$

The functions $K_\pm(\alpha)$ are defined by

$$K_\pm(\alpha) = \frac{(\alpha \pm \gamma)g_\pm(\alpha)}{\sqrt{\beta}(\alpha \pm \alpha_0)(\alpha \pm \alpha_{-1})(\alpha \pm \alpha_{-2})}.$$

In this case, $K_+(\alpha) = K_-(-\alpha)$.

Using the representation

$$e^{i\alpha x_*} K_-(\alpha) = L_+(\alpha) + L_-(\alpha), \qquad L_\pm(\alpha) = \pm\frac{1}{2\pi i}\int\limits_{-\infty\mp i\sigma}^{\infty\mp i\sigma} \frac{e^{i\zeta x_*} K_-(\zeta)\, d\zeta}{\zeta - \alpha}, \qquad \sigma < |\gamma_1|, c, \tag{2.5}$$

we write Eq. (2.4) as

$$K_-(\alpha)F_-(\alpha) - iL_-(\alpha) = D_+(\alpha)/K_+(\alpha) + iL_+(\alpha).$$

The left side of this equality contains a function analytic in the regions S_-, and the right side a function analytic in S_+. An analytic continuation of these functions gives a function analytic over the entire complex plane. According to Liouville's theorem, this function is a polynomial. The polynomial degree is determined by the behavior of the functions as $|\alpha| \to \infty$.

The condition of local boundedness of energy implies that near the plate edge, the velocities have a singularity not higher than $O(r^{-\lambda})$ ($\lambda < 1$; r is the distance to the plate edge). Then, for $|\alpha| \to \infty$, the function $F_-(\alpha)$ has order not higher than $O(|\alpha|^{\lambda+3})$ and the order of $D_+(\alpha)$ is not higher than $O(|\alpha|^{\lambda-1})$ [10]. At infinity, the functions $K_\pm(\alpha)$ have order $O(|\alpha|^{-2})$ since $g^\pm(\alpha) \to 1$ as $|\alpha| \to \infty$. It is easy to show that $|L_\pm(\alpha)| = O(|\alpha|^{-1})$ for $|\alpha| \to \infty$. Hence, the polynomial degree is equal to unity and

$$D_+(\alpha)/K_+(\alpha) + iL_+(\alpha) = i(a + b\alpha),$$

where a and b are unknown constants, which should be determined from conditions (1.6).

Expressing $D_+(\alpha)$ from the last equation and taking into account (2.2) and (2.3), we obtain

$$\varphi(x, y) = -\frac{1}{2\pi i}\int\limits_{-\infty}^{\infty} \frac{e^{-i\alpha x} \cosh\,(\alpha(y + H))K_+(\alpha)}{\cosh\,(\alpha H)K_1(\alpha)} \Big(a + b\alpha - L_+(\alpha)\Big)\, d\alpha. \tag{2.6}$$

The contour of integration should be chosen so that it entirely lies in the intersection of the regions S_+ and S_-. The contour of integration on the real axis can be chosen so as to bypass the points α_0 and γ from below and the points $-\alpha_0$ and $-\gamma$ from above.

Let us consider the case $x > 0$. The integral is calculated using residue theory. Expression (2.6) is written as

$$\varphi(x, y) = -\frac{1}{2\pi i}\int\limits_{-\infty}^{\infty} \frac{e^{-i\alpha x} \cosh\,(\alpha(y + H))}{K_-(\alpha)K_2(\alpha) \cosh\,(\alpha H)} \Big(a + b\alpha + L_-(\alpha)\Big)\, d\alpha + \frac{1}{2\pi i}\int\limits_{-\infty}^{\infty} \frac{e^{-i\alpha(x - x_*)} \cosh\,(\alpha(y + H))\, d\alpha}{K_2(\alpha) \cosh\,(\alpha H)}. \tag{2.7}$$

For the first integral, the contour of integration is closed in the lower semiplane, and for the second integral, it is closed in the upper half plane. For $x > 0$, we obtain

$$\frac{\partial\varphi}{\partial y}(x, 0) = \sum_{j=-2}^{\infty} \frac{e^{i\alpha_j x}\, \alpha_j \tanh\,(\alpha_j H)}{K_-(-\alpha_j)K_2'(-\alpha_j)} \Big(a - b\alpha_j + L_-(-\alpha_j)\Big) + \sum_{j=-2}^{\infty} \frac{e^{i\alpha_j |x - x_*|}\, \alpha_j \tanh\,(\alpha_j H)}{K_2'(\alpha_j)}. \tag{2.8}$$

From the dispersion relation for the region under the plate, we have

$$\alpha_j \tanh\,(\alpha_j H) = -K_1(\alpha_j)/(\beta\alpha_j^4 - \delta).$$

Substitution of this expression into formula (2.8) and then into boundary conditions (1.6) yields the following system of second-order linear algebraic equations for the unknowns a and b:

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}. \tag{2.9}$$

According to the residue theorem, the coefficients of the system can be written as

$$A_{11} = \sum_{k=1}^{4} \mathrm{res}_{z_k} \Big(\frac{\alpha^2 K_+(\alpha)}{\beta\alpha^4 - \delta}\Big), \qquad A_{12} = A_{21},$$

$$A_{21} = \sum_{k=1}^{4} \mathrm{res}_{z_k} \Big(\frac{\alpha^3 K_+(\alpha)}{\beta\alpha^4 - \delta}\Big), \qquad A_{22} = \sum_{k=1}^{4} \mathrm{res}_{z_k} \Big(\frac{\alpha^4 K_+(\alpha)}{\beta\alpha^4 - \delta}\Big),$$

$$C_1 = -\sum_{k=1}^{4} \mathrm{res}_{z_k} \Big(\frac{\alpha^2 K_+(\alpha) L_-(\alpha)}{\beta\alpha^4 - \delta}\Big), \qquad C_2 = -\sum_{k=1}^{4} \mathrm{res}_{z_k} \Big(\frac{\alpha^3 K_+(\alpha) L_-(\alpha)}{\beta\alpha^4 - \delta}\Big),$$

where z_k are the roots of the polynomial $\beta\alpha^4 - \delta = 0$. From (2.5), we obtain

$$L_-(\alpha) = -\sum_{j=-2}^{\infty} \frac{\mathrm{e}^{i\alpha_j x_*} K_1(\alpha_j)}{K_2'(\alpha_j) K_+(\alpha_j)(\alpha_j - \alpha)}.$$

The coefficients of the system are converted as follows:

$$A_{11} = \sum_{k=1}^{4} \frac{K_+(z_k)}{z_k}, \qquad A_{12} = A_{21} = \sum_{k=1}^{4} K_+(z_k), \qquad A_{22} = \sum_{k=1}^{4} z_k K_+(z_k),$$

$$C_1 = -\sum_{k=1}^{4} \frac{K_+(z_k) L_-(z_k)}{z_k}, \qquad C_2 = -\sum_{k=1}^{4} K_+(z_k) L_-(z_k).$$

Determining the coefficients a and b from system (2.9) and substituting them into formulas (2.6) and (2.7), we find all the desired quantities. For the amplitude of elevation of the free boundary at infinity η_∞,

$$\eta_\infty = \Big|\frac{K_+(\gamma)}{K_1'(\gamma)} \left[(a + b\gamma - L_+(\gamma)\right]\Big|. \tag{2.10}$$

From (1.1), the amplitude of plate deflection is defined by the relation $|w(x)| = |\varphi_y(x, 0)|$ and expression (2.8). In expression (2.8), the second term represents the waves propagating from the load application point and coincides with the deflection of an infinite plate and the first term represents the waves reflected from the edge. The amplitude of the plate deflection at infinity is calculated by the formula

$$w_\infty = \frac{\alpha_0 \tanh{(\alpha_0 H)}}{K_2'(\alpha_0)} \Big|\mathrm{e}^{-i\alpha_j x_*} - \frac{a - b\alpha_0 + L_-(-\alpha_0)}{K_+(\alpha_0)}\Big|.$$

3. Solution Ignoring the Draft. According to the above assumptions, $\delta \ll 1$; therefore, in Eq. (1.5), the parameter δ can be neglected. As $\delta \to 0$, we have

$$A_{11} = K_+'(0), \qquad A_{12} = A_{21} = K_+(0), \qquad A_{22} = 0,$$

$$C_1 = -(K_+(0) L_-(0))', \qquad C_2 = -K_+(0) L_-(0).$$

Then, we obtain

$$a = -L_-(0) = \sum_{j=-2}^{\infty} \frac{\mathrm{e}^{i\alpha_j x_*} K_1(\alpha_j)}{K_2'(\alpha_j) K_+(\alpha_j)\alpha_j}, \qquad b = -L_-'(0) = \sum_{j=-2}^{\infty} \frac{\mathrm{e}^{i\alpha_j x_*} K_1(\alpha_j)}{K_2'(\alpha_j) K_+(\alpha_j)\alpha_j^2}.$$

Substituting the coefficients a and b into (2.10) and using the relation

$$|K_+(\gamma)| = \sqrt{\frac{2\gamma(\gamma - \alpha_0)|K_1'(\gamma)|}{(\gamma + \alpha_0)|K_2(\gamma)|}},$$

we obtain

$$\eta_\infty = \sqrt{\frac{2\gamma(\gamma - \alpha_0)}{(\gamma + \alpha_0)\beta K_1'(\gamma)}} \Big|\sum_{j=-2}^{\infty} \frac{\mathrm{e}^{i\alpha_j x_*} K_1(\alpha_j)}{K_2'(\alpha_j) K_+(\alpha_j)\alpha_j^2(\alpha_j - \gamma)}\Big|.$$

From (2.8), we have

$$w(x) = i\sum_{j=-2}^{\infty} \frac{\alpha_j \tanh{(\alpha_j H)}}{K_2'(\alpha_j)} \Big[\mathrm{e}^{i\alpha_j|x - x_*|} + \frac{\alpha_j^2 \mathrm{e}^{i\alpha_j x}}{K_+(\alpha_j)} \sum_{n=-2}^{\infty} \frac{\mathrm{e}^{i\alpha_n x_*} K_1(\alpha_n)}{K_2'(\alpha_n) K_+(\alpha_n)\alpha_n^2(\alpha_n + \alpha_j)}\Big].$$

We next consider the case of a distributed load. Let the plate subjected to a time-periodic pressure of intensity $q(x)$, $x \in [x_1, x_2]$. In this case, multiplying the solution obtained by $q(x_*)$ and integrating over x_*, we find

$$\eta_\infty = \sqrt{\frac{2\gamma(\gamma - \alpha_0)}{(\gamma + \alpha_0)\beta K_1'(\gamma)}} \Big|\sum_{j=-2}^{\infty} \frac{A_j K_1(\alpha_j)}{K_2'(\alpha_j) K_+(\alpha_j)\alpha_j^2(\alpha_j - \gamma)}\Big|,$$

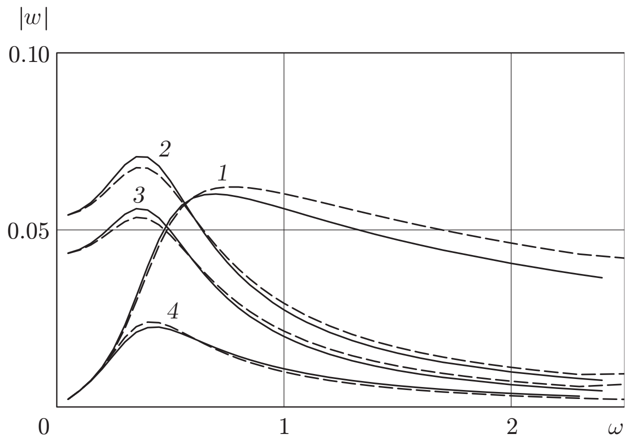

Fig. 1

$$w(x) = i \sum_{j=-2}^{\infty} \frac{\alpha_j \tanh(\alpha_j H)}{K_2'(\alpha_j)} \Big[B_j(x) + \frac{\alpha_j^2 \, e^{i\alpha_j x}}{K_+(\alpha_j)} \sum_{n=-2}^{\infty} \frac{A_n K_1(\alpha_n)}{K_2'(\alpha_n) K_+(\alpha_n) \alpha_n^2 (\alpha_n + \alpha_j)} \Big],$$

$$B_j(x) = \int_{x_1}^{x_2} e^{i\alpha_j |x - x_*|} q(x_*)\, dx_*, \qquad A_j = \int_{x_1}^{x_2} e^{i\alpha_j x_*} q(x_*)\, dx_*,$$

$$w_\infty = \frac{\alpha_0 \tanh(\alpha_0 H)}{K_2'(\alpha_0)} \Big| \bar{A}_0 + \frac{\alpha_0^2}{K_+(\alpha_0)} \sum_{n=-2}^{\infty} \frac{A_n K_1(\alpha_n)}{K_2'(\alpha_n) K_+(\alpha_n) \alpha_n^2 (\alpha_n + \alpha_0)} \Big|$$

(the bar denotes complex conjugation).

4. Numerical Results. Calculations were performed for a semi-infinite ice plate in ocean for the following parameters: $E = 6 \cdot 10^9$ N/m^2, $\rho = 1025$ kg/m^3, and $\rho_0 = 922.5$ kg/m^3. The dependence of the external load on the plate on x was given by

$$q(x) = \begin{cases} q_0[1 - (x - x_0)^2/d^2], & |x - x_0| < d, \\ 0, & |x - x_0| > d, \end{cases}$$

where d is the half-width of the pressure region and x_0 is the center of load application. In this case,

$$A_j = \frac{4}{\alpha_j^2 d} \Big[\frac{\sin(\alpha_j d)}{\alpha_j d} - \cos(\alpha_j d) \Big].$$

The plate thickness, liquid depth, and the loading frequency, center, and area were varied.

The calculations showed that each of the above-mentioned parameters is important in the problem at hand. Figure 1 shows the solution taking into account the draft (solid curves) and the solution ignoring it (dashed curves) for $H_0 = 100$ m, $x_0 = 20$ m, $d = 2$ m, and $h = 5$ m. Curves 1 correspond to the amplitude of elevation of the free surface at infinity η_∞, curves 2 to the amplitude of plate deflection at the edge $|w(0)|$, curves 3 to the amplitude at the center of load application $|w(x_0)|$, and curves 4 to the amplitude of the outgoing wave in the plate w_∞. From Fig. 1 it follows that the parameter δ has a more significant effect on the amplitude of elevation of the free surface at high frequencies than on the amplitude of plate deflection. The plate vibration amplitudes are small under high-frequency external actions. Solutions in explicit form ignoring the draft can be used to estimate the plate deflection.

The effect of liquid depth on the vibration amplitudes is significant for low depths and frequencies. As the depth increases, the solution rapidly reaches the asymptotic form for an infinitely deep liquid, and then it is little affected by depth variation. The effect of the liquid depth on the vibration amplitude is more significant for thick plates. Figure 2 shows frequency dependences of the amplitudes of free-surface elevation at infinity (curves 1), plate deflection at the edge (curves 2), plate deflection at the center of load application (curves 3), and the frequency dependence of the amplitude of the outgoing wave in the plate at infinity (curves 4) for $h = 1.5$ m, $x_0 = 20$ m, and

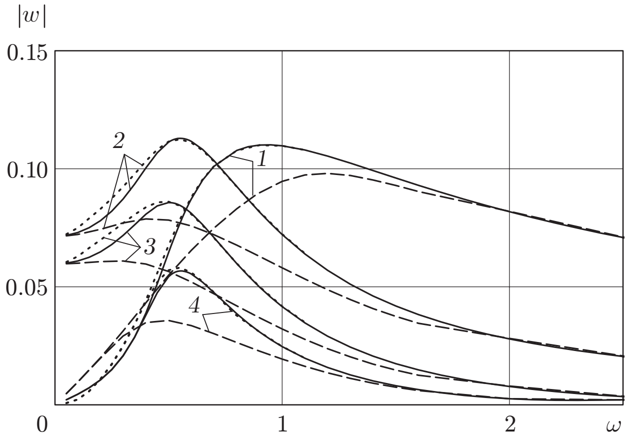

Fig. 2

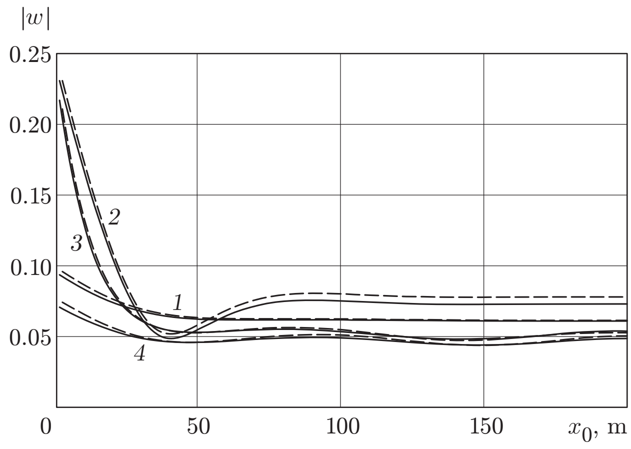

Fig. 3

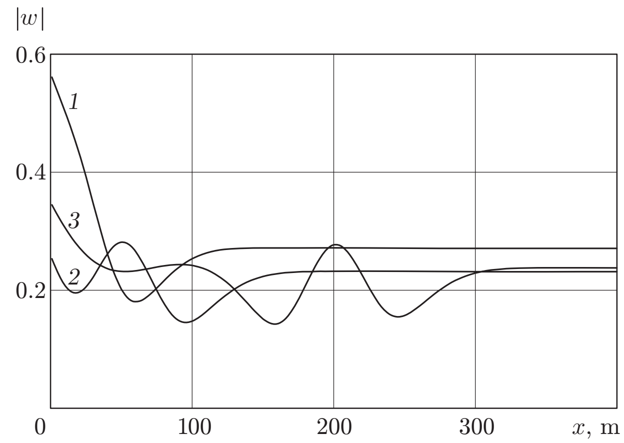

Fig. 4

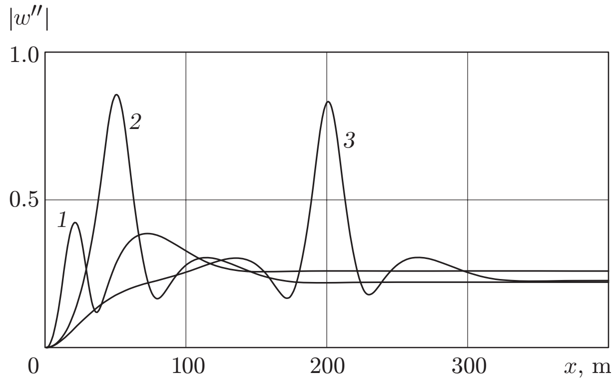

Fig. 5

$d = 2$ m. The solid curves correspond to a depth $H_0 = 100$ m, dashed curves to $H_0 = 20$ m, and dotted curves to $H_0 = 1000$ m. For depths $H_0 = 100$ and 1000 m, the difference between the curves is small and is observed only for small frequencies. From Figs. 1 and 2, it is evident that the vibration amplitudes of the free surface and the plate depend nonmonotonically on the frequency. At a certain frequency, the amplitudes reach maxima and then decrease as the frequency increase. The frequency corresponding to the maximum amplitude depends on all parameters of the problem. In all calculations, it did not exceed 1 $\sec^{-1}$.

The maximum amplitudes of plate deflection are observed at the point of maximum load or at the plate edge. Variation in the position of the center of load application on the plate can change their ratio. The plate strain amplitudes are maximal at the center of load application. Figure 3 shows curves of the vibration amplitudes of the liquid and the plate versus the position of the center of load application at $\omega = 0.5$ $\sec^{-1}$, $h = 1.5$ m, $d = 2$ m, and $H_0 = 100$ m. The notation of the curves is same as in Figs. 1 and 2. Solid curves correspond to the complete solution, and dashed to the solution ignoring draft. It is obvious that at large distances from the edge, the amplitudes of the surface waves η_∞ and the plate edge $|w(0)|$ are stabilized, and the deflection amplitudes at the center of load application $|w(x_0)|$ and at infinity w_∞ vary under a harmonic law. This also follows from the formulas obtained.

Figure 4 gives the plate deflection amplitudes, and Fig. 5 shows the dimensionless strain amplitudes $|w''(x)|$ for $\omega = 0.5$ $\sec^{-1}$, $h = 1.5$ m, $d = 10$ m, and $H_0 = 100$ m for various positions of the center of loading [$x_0 = 20$ (1), 50 (2), and 200 m (3)]. As the area of load application increases, the vibration amplitude of the liquid and the plate vary nonmonotonically. The vibration level for the liquid and the plate is basically determined by the quantity $|A_0|$. Indeed, the quantity $|A_0|$ determines the amplitude of the wave propagating in the plate from the region of load

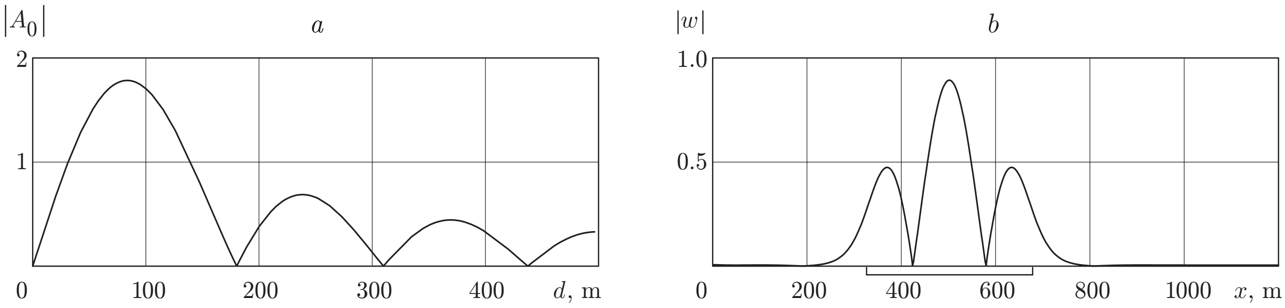

Fig. 6

application and the amplitude of the waves reflected from the edges. The dependence of $|A_0|$ on the area of load application d for $\omega = 0.5$ sec^{-1}, $h = 1.5$ m, and $H_0 = 100$ is presented in Fig. 6a. The vibration amplitudes of the free surface, the plate at infinity, and at the plate edge vary in proportion to this quantity. If the region of load application is away from the plate edges, then $|A_j| \ll 1$ for $j \neq 0$. Thus, if $A_0 = 0$, the plate vibrations are same as for an infinite plate: standing waves are concentrated near the region of load application, and the remaining part of the plate and the liquid are nearly undisturbed. Figure 6b shows an example of such vibrations for $\omega = 0.5$ sec^{-1}, $h = 1.5$ m, $d = 180$ m, $H_0 = 100$ m, and $x_0 = 500$ m. If $A_0 \neq 0$, waves reflected from the edges always exist. Thus, the frequency and nature of the load-intensity distribution have a significant effect on the amplitudes of surface waves and plate deflection.

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